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Precursor inter-symbol interference removal by block transmission-based time-reversed equalization

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Abstract

Single-carrier transmission is considered in the general finite impulse response inter-symbol interference (ISI) channel. In an ISI channel with a matched filter, the folded spectrum of the received pulse can be factored into a minimum phase causal part and a maximum phase anticausal part corresponding to the postcursor and precursor ISI, respectively. In this paper, zero-forcing ISI cancellation is considered. In a direct implementation, the precursor equalization is carried out based on truncating and delaying the ideal anticausal precursor equalizer impulse response. In the proposed scheme, a block transmission is adopted, and the precursor equalization is carried out by a time reversal within each block and using a practical minimum phase filter. We show that the ISI can be removed perfectly using the proposed scheme. By means of a numerical example, it is shown that the proposed scheme achieves improved performance compared to the truncate- and delay-based equalizer in terms of transmission rate, delay, and implementation complexity.

Keywords: Equalization; Inter-symbol interference; Spectral factorization; Zero-forcing equalizer

1 Introduction

Consider the single-carrier transmission system and its discrete-time equivalent shown in Figure 1a,b, where T is the symbol duration and $h(t)$ denotes the received pulse (or the overall transmit filter and channel impulse response) with duration T_h. Let

$$\rho_\mathrm{h}[n] = \int_{-\infty}^{\infty} h(t)h^*(t-nT)\,\mathrm{d}t \tag{1}$$

denote the sampled autocorrelation function of $h(t)$ at time n, where $*$ denotes complex conjugate operation. The parameter $E_\mathrm{h} \equiv \rho_\mathrm{h}[0]$ is the received pulse energy, $\rho_\mathrm{h}[n]$ for $n > 0$ is called the postcursor inter-symbol interference (ISI), i.e., the ISI from past data symbols, and $\rho_\mathrm{h}[n]$ for $n < 0$ is called the precursor ISI, i.e., the ISI from future data symbols [1]. The folded spectrum of $h(t), S_\mathrm{h}(z)$, is defined as the z-transform of $\rho_\mathrm{h}[n]$. Let

$$S_\mathrm{h}(z) = \gamma^2 M(z)M^*\left(1/z^*\right) \tag{2}$$

denote the spectral factorization of $S_\mathrm{h}(z)$, where $M(z)$ is monic (i.e. $\mu[0] = 1$) and minimum phase with $\mu[n]$ being its time domain representation. As a result, $M^*(1/z^*)$ is monic and maximum phase [1]. The spectral factorization of $S_\mathrm{h}(z)$ can be obtained by Kolmogorov 1939 approach [2].

A zero-forcing equalizer consists of a postcursor equalizer and a precursor equalizer, a.k.a. forward equalizer. The postcursor and precursor equalizers remove the postcursor and precursor ISI, respectively. The system function of the postcursor equalizer is given by [1]:

$$E(z) = M^{-1}(z). \tag{3}$$

Since $M(z)$ is minimum phase, $E(z)$ can be implemented easily in practice[a] [1]. Similarly, the system function of the precursor equalizer is given by

$$D(z) = \left(M^*\left(1/z^*\right)\right)^{-1}. \tag{4}$$

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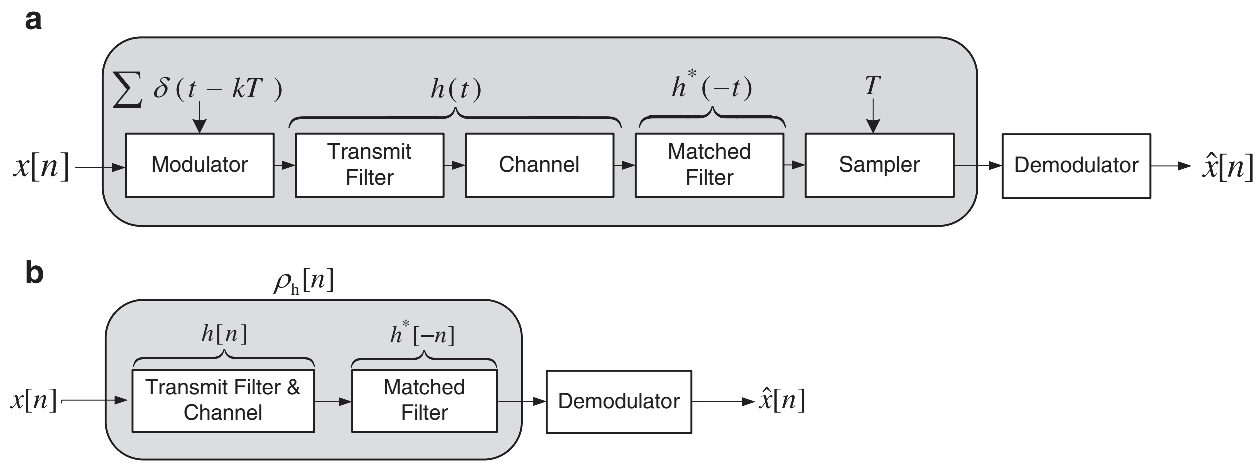

Figure 1 A schematic of the single-carrier transmission system: (a) continuous-time model and (b) discrete-time model.

Since $M(z)$ is minimum phase, $M^*(1/z^*)$ and $D(z)$ are maximum phase. Therefore, for $D(z)$ to be stable, it must be anticausal, meaning that it cannot be implemented in the general case. Only when $D(z)$ is finite impulse response (FIR) that it can be implemented as a causal filter by introducing some delay [1]. It is easy to show that when the postcursor and precursor equalizers are implemented perfectly, the transfer function of the entire system is the constant γ^2, and therefore, the ISI is removed perfectly. Unfortunately, $D(z)$ is not FIR when the received pulse has a finite duration T_h, because the inverse of a FIR system is infinite impulse response (IIR)[b]. A multipath channel is an example of a FIR channel, and together with an FIR transmit filter, it leads to an FIR received pulse and hence an IIR $D(z)$. An example of a system with a FIR transmit filter is the filtered multi-tone (FMT) modulation [3]. Conventionally, an IIR anticausal filter is approximated by truncating and delaying in time leading to a causal FIR filter [1]. In this paper, we call this scheme the truncate and delay (T&D) scheme.

In this paper, we propose an alternative scheme to address this problem, namely block transmission-based time-reversed equalization (BT-RTE). The basic idea is to adopt block transmission and implement the precursor equalizer by a time reversal within each block and using a practical minimum phase filter. The idea of using block transmission and reversing the received symbol stream in each block in equalization has been considered in a few articles before. In [4], a decision feedback equalizer (DFE) is operated on a time-reversed stream in order to achieve a better performance in maximum phase channels. In [5], bidirectional DFE has been proposed in which two DFE's operate on the received block and the time-reversed received block. Then, the two outputs are compared, and if the decoded bits are different for the two streams (i.e., conflicting decisions), the more likely bit is chosen comparing the corresponding Euclidean metrics. In [6], an improved receiver structure is proposed by trellis-based conflict resolution. Time reversal is also used in the context of antenna arrays [7], multiple-input multiple-output systems [8], and space-time block coding [9].

The techniques proposed in [4,5] and [6] do not consider matched filtering. Therefore, precursor ISI cancellation is not considered. Focusing on linear zero-forcing equalizers, in this paper we consider matched filtering and propose a simple linear equalizer to remove precursor ISI[c]. We show that the ISI can be removed perfectly using the proposed scheme. By means of a numerical example, we show that the proposed scheme achieves improved performance compared to the T&D-based equalizer in terms of transmission rate, delay, and implementation complexity.

2 Block transmission-based time-reversed equalization

A block diagram of the proposed BT-RTE scheme is depicted in Figure 2, where the modulator, transmit filter, channel, matched filter, and sampler are replaced by their equivalent discrete-time model, i.e., a discrete-time filter with impulse response $\rho_h[n]$. As it can be seen, the transmitted data symbols are sent in blocks at the transmitter side. At the receiver side, each block is reversed in time, fed through a filter with system transfer function $(M^*(z^*))^{-1}$ and then it is reversed back in time. Note that since $M(z)$ is minimum phase, $M^*(z^*)$ is also minimum phase and hence $(M^*(z^*))^{-1}$ is practical. This three-step procedure is in fact a practical realization of the precursor equalizer $(M^*(1/z^*))^{-1}$. Then, the signal is fed through the postcursor equalizer with system function $M^{-1}(z)$ to obtain the equalized data symbols.

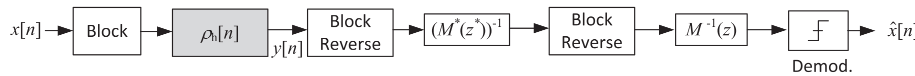

Figure 2 The proposed equalization scheme.

It is easy to show that the proposed three-step procedure is equivalent to a filter with the desired precursor equalizer system function $(M^*(1/z^*))^{-1}$. Let $y[n]$ denote the signal at the output of the matched filter with z-transform $Y(z)$. The z-transform for the reversed signal $y[-n]$ is $Y(1/z)$. The z-transform of the output of the filter with transfer function $(M^*(z^*))^{-1}$ is $(M^*(z^*))^{-1}\, Y(1/z)$. Finally, the z-transform of the signal when it is reversed back in time is $(M^*(1/z^*))^{-1}\, Y(z)$. For this to work properly, we need to avoid inter-block interference (IBI), which can be done by adding a guard interval with length equal to the total transmit filter and channel length[d], at the end of each block[e]. In this paper, we assume that the transmission is idle during the guard time. Another approach is the use of a cyclic prefix during the guard time [10].

3 Numerical example and analysis

In this section, we study a practical example with a two-tap FIR channel. Let

$$h(t) = g(t) - cg(t - T) \tag{5}$$

denote the channel impulse response, where $g(t)$ is a real-valued pulse with unit energy and duration T, c is a complex-valued constant, and T is the symbol duration. The discrete model of the channel is then $h[n] = \delta[n] - c\delta[n-1]$. The matched filter's impulse response for this channel is $h^*(-t) = g(-t) - c^* g(-t-T)$ or in the discrete domain $h^*[-n] = \delta[n] - c^*\delta[n+1]$ with z-transform $H^*(1/z^*) = 1 - c^* z$, where $H(z) = 1 - cz^{-1}$ is the matched filter transfer function. In practice, we have to delay the matched filter by $2T$ (or one sample in discrete domain) to obtain a causal filter, which following [1] we ignore here. Therefore, $\rho_{\mathrm{h}}[n] = \left(1 + |c|^2\right)\delta[n] - c^*\delta[n+1] - c\delta[n-1]$ and $S_{\mathrm{h}}(z) = 1 + |c|^2 - c^* z - cz^{-1}$.

The spectral factorization of $S_{\mathrm{h}}(z)$ depends on the amplitude of c. Assuming $|c| < 1$, we obtain $M(z) = 1 - cz^{-1}$. Thus, $E(z) = \left(1 - cz^{-1}\right)^{-1}$ and $D(z) = (1 - c^* z)^{-1}$. For the postcursor and precursor equalizers to be stable, the region of convergence (ROC) should include the unit circle [11]. Since $|c| < 1$, stability necessitates the ROC to be $|z| > |c|$ for the postcursor equalizer $E(z)$. Moreover, the impulse response of the postcursor equalizer is calculated to be $e[n] = c^n u[n]$ (where $u[n]$ is the unit step function), which is a causal filter and can be implemented using a feedback loop with open loop gain $1 - M(z)$ [1]. For $D(z)$, the stability requirement leads to the ROC $|z| < |c|^{-1}$. Therefore, the impulse response is calculated to be $d[n] = c^{*-n} u[-n]$, which is an anticausal filter. Using similar reasoning, for $|c| > 1$, we obtain $M(z) = 1 - c^* z, E(z) = (1 - c^* z)^{-1}, D(z) = \left(1 - cz^{-1}\right)^{-1}$, $e[n] = -c^{*-n-1} u[n-1]$, and $d[n] = -c^n u[-n-1]$. Note that $d[n]$ is again anticausal.

3.1 T&D equalizer

Since $d[n]$ is an infinite length anticausal filter, it is not practically implementable and can only be approximated based on a T&D operation [1]. Let us first consider the $|c| < 1$ case. If we truncate $d[n]$ at $n = -L$ (i.e., set $d[n] = 0$ for $n \le -L$) and delay it by $L - 1$ samples, we obtain the following L-tap FIR approximation for the precursor equalizer:

$$\hat{d}[n] = c^{*L-1-n}\left(u[n] - u[n-L]\right). \tag{6}$$

The z-transform of $\hat{d}[n]$ is

$$\hat{D}(z) = c^{*L-1}\frac{1 - (c^* z)^{-L}}{1 - (c^* z)^{-1}} = c^{*L} z \frac{(c^* z)^{-L} - 1}{1 - c^* z}. \tag{7}$$

Thus, the overall system transfer function is

$$S_{\mathrm{h}}(z)\hat{D}(z)E(z) = c^{*L} z\left((c^* z)^{-L} - 1\right) = z^{-L+1} - c^{*L} z, \tag{8}$$

and its impulse response is $h_{\mathrm{s}}[n] = \delta[n-L+1] - c^{*L}\delta[n+1]$. Therefore, the output of the system to the input $x[n]$ is $s[n] = x[n-L+1] - c^{*L} x[n+1]$. As it can be seen, the system is not causal which is because the matched filter is not causal. Note that $x[n-L+1]$ is the desirable signal (delayed by $L-1$ samples), and $-c^{*L} x[n+1]$ is the residual ISI due to the approximation in the precursor equalizer. If $P \equiv \mathrm{E}\left\{|x[n]|^2\right\}$, then the desired signal and ISI power at the output of the postcursor equalizer are P and $P|c|^{2L}$, respectively.

We assume that the channel noise is white and Gaussian with variance σ^2. The noise goes through the matched filter and the equalizers with in total the following transfer function

$$H_{\mathrm{n}}(z) = c^{*L} z \frac{(c^* z)^{-L} - 1}{1 - cz^{-1}}. \tag{9}$$

Therefore, if we denote the channel noise by $\phi[n]$, the noise contribution at the output of the postcursor equalizer is

$$\nu[n] = -c^{*L} \sum_{m=0}^{L-1} c^m \phi\left[n-m-1\right] + c^{*L} \sum_{m=L}^{\infty} c^m \left(|c|^{-2L}-1\right) \times \phi\left[n-m-1\right]. \tag{10}$$

From this the noise power is calculated to be

$$\mathrm{E}\left\{|\nu[n]|^2\right\} = \sigma^2 \frac{1-|c|^{2L}}{1-|c|^2}. \tag{11}$$

Thus the achievable signal-to-interference-and-noise ratio (SINR) for the T&D equalizer is

$$\mathrm{SINR}_{\mathrm{T\&D}} = \left[|c|^{2L} + \mathrm{SNR}_\circ^{-1}\left(1+|c|^2\right)\frac{1-|c|^{2L}}{1-|c|^2}\right]^{-1}, \tag{12}$$

where

$$\mathrm{SNR}_\circ \equiv \left(1+|c|^2\right)\frac{P}{\sigma^2} \tag{13}$$

is the SNR of the channel when ISI is not present, i.e., when the previous symbol is zero. Finally, assuming complex symbols with Gaussian distribution, the number of bits that can be transmitted per channel use is obtained by the Shannon capacity formula

$$C_{\mathrm{T\&D}} = \log_2\left(1+\mathrm{SINR}_{\mathrm{T\&D}}\right). \tag{14}$$

For the $|c| > 1$ case, we truncate $d[n]$ at $n = -L-1$ and delay it by L samples to obtain

$$\hat{d}[n] = -c^{n-L}\left(u[n] - u\left[n-L\right]\right), \tag{15}$$

with z-transform

$$\hat{D}(z) = -c^{-L}\frac{1-c^L z^{-L}}{1-cz^{-1}}. \tag{16}$$

The overall system transfer function and impulse response are $S_{\mathrm{h}} = z^{-L} - c^{-L}$ and $h_{\mathrm{s}} = \delta\left[n-L\right] - c^{-L}\delta[n]$. The system transfer function for the noise is

$$H_{\mathrm{n}}(z) = -c^{-L}\frac{1-c^L z^{-L}}{1-cz^{-1}}, \tag{17}$$

and the noise power is equal to

$$\mathrm{E}\left\{|\nu[n]|^2\right\} = \sigma^2 |c|^{-2}\frac{1-|c|^{-2L}}{1-|c|^{-2}}. \tag{18}$$

Finally, the achievable SINR is

$$\mathrm{SINR}_{\mathrm{T\&D}} = \left[|c|^{-2L} + \mathrm{SNR}_\circ^{-1}\left(1+|c|^{-2}\right)\frac{1-|c|^{-2L}}{1-|c|^{-2}}\right]^{-1}, \tag{19}$$

which is equal to (12) with c replaced by c^{-1}. The number of bits per channel use can be obtained by (14).

3.2 BT-RTE

Now, let us analyze the performance of the BT-RTE for this channel. Using the BT-RTE, the ISI is compensated perfectly and the overall transfer function is 1. The channel noise goes through the matched filter and the pre- and postcursor equalizers. It is easy to show that the system transfer function for the channel noise for both $|c| < 1$ and $|c| > 1$ cases is $H^{-1}(z) = \left(1-cz^{-1}\right)^{-1}$; however, the noise power is different for the two cases as the pre- and postcursor equalizers are implemented differently as discussed before. For the $|c| < 1$ case, the noise power for the n-th symbol in the block is calculated by

$$\mathrm{E}\left\{|\nu[n]|^2\right\} = \sigma^2\frac{1-|c|^{2n}}{1-|c|^2}. \tag{20}$$

For the $|c| > 1$ case, the noise power is calculated by

$$\mathrm{E}\left\{|\nu[n]|^2\right\} = \sigma^2 |c|^{-2}\left[|c|^{-2(n+2)}\frac{1-|c|^{-2n}}{1-|c|^{-2}} + \left|1-|c|^{-2(n+1)}\right|^2\frac{1-|c|^{-2(K-n)}}{1-|c|^{-2}}\right], \tag{21}$$

where K is the block length. The achievable SNR (or SINR) on the n-th received symbol using the BT-RTE is

$$\mathrm{SNR}_{\mathrm{BT\text{-}RTE},n} = f_n^{-1}\mathrm{SNR}_\circ, \tag{22}$$

where

$$f_n \equiv \left(1+|c|^2\right)\frac{\mathrm{E}\left\{|\nu[n]|^2\right\}}{\sigma^2} \tag{23}$$

is the noise power boost factor on the n-th symbol due to the equalization.

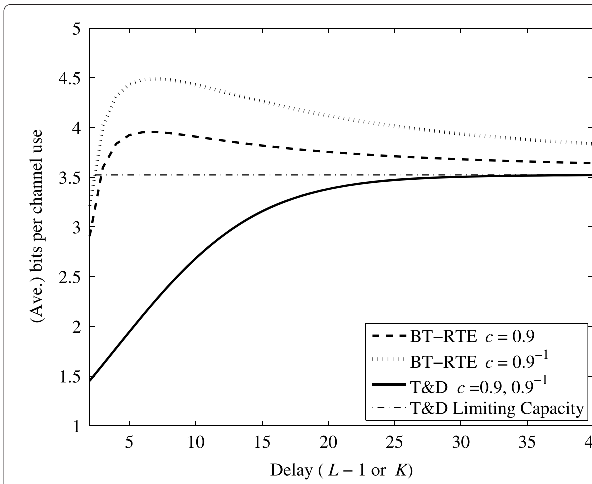

Figure 3 The (average) achievable transmission bits per channel use vs. delay. The (average) achievable transmission bits per channel use for the T&D equalizer and the BT-RTE scheme vs. delay for the channel described by (5) for 0.9 and $c = 0.9^{-1}$ and $\mathrm{SNR}_\circ = 20$ dB.

For the BT-RTE scheme, we need a guard interval of length one in each block for this channel to avoid IBI. Assuming symbol by symbol detection, the average number of transmitted bits per channel use is calculated by

$$C_{\text{BT-RTE}} = \frac{1}{K}\sum_{n=1}^{K-1}\log_2\left(1+\text{SNR}_{\text{BT-RTE},n}\right). \tag{24}$$

The (average) number of transmitted bits per channel use for the two schemes is plotted vs. the delay in Figure 3 for $c = 0.9$ and 0.9^{-1} and $\text{SNR}_\circ = 20$ dB. Note that the delay for the T&D equalizer is $L-1$, and the maximum delay of the symbols in a block is K for the BT-RTE scheme. As it can be seen, the achievable rate for the BT-RTE scheme is significantly higher than for the T&D equalizer for $c = 0.9$ and 0.9^{-1}, i.e., when the channel zero is close to the unit circle. In fact, the T&D scheme is an approximation of the zero-forcing equalizer (ZFE), which is known to boost the channel noise in this case [12]. On the other hand, the BT-RTE is a perfect implementation of the ZFE; however, it does not boost the noise power outside the block using a guard time.

Moreover, it can be noticed that the BT-RTE scheme's achievable rate is not always an increasing function of the block size (or delay) K. In fact, the peak point is located at $K = 7$, which is about 27.48% higher than the limiting capacity of the T&D scheme, and a block length of $K = 3$ is enough to reach the limiting capacity. This is because the loss due to the guard interval decreases by increasing the block length K. However, the SNR also decreases by increasing n $(1 \leq n < K)$ in (22). As a result, the average rate is not necessarily an increasing function of K. If we ignore the loss due to the guard time, the achievable bit rate is always a monotonically decreasing function of the block length approaching the limiting capacity of the T&D scheme in limit.

Finally, note that the computational complexity of the BT-RTE scheme is independent of the block size K and depends on the channel impulse response length. However, the computational complexity of the T&D scheme grows almost linearly with the filter length L.

4 Conclusion

Precursor ISI equalizers are not realizable in many situations and can only be approximated by truncating and delaying the ideal (anticausal) filter impulse response. In this paper, we have proposed a block transmission scheme in which the precursor equalizer is implemented in reversed time. We have shown that this filter is minimum phase and practically implementable. By means of a numerical example, we have shown that the proposed system can achieve limiting rates with smaller delays and lower computational complexity compared to a scheme based on truncating and delaying when the zeros of the channel are close to the unit circle.

Endnotes

[a] We assume that $M(z)$ does not have a zero on the unit circle.

[b] Indeed the inverse of rational IIR systems is also IIR except for all-pole IIR systems.

[c] Nevertheless, a higher performance can be achieved by the use of minimum mean squared equalizer (MMSE), decision feedback equalizer, or maximum likelihood (ML) decoding instead of linear zero-forcing equalization.

[d] In discrete time, the length of the guard interval is the overall transmit filter and channel impulse response length minus one.

[e] For this to be valid, the blocks should be separated at the output of the channel; otherwise, the matched filter length should also be added to the length of the guard interval.

Competing interests

The authors declare that they have no competing interests.

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The scientific responsibility is assumed by its authors.

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References

1. JR Barry, EA Lee, DG Messerschmitt, *Digital Communication*, 3rd edn. (Kluwer, Boston, 2003)
2. A Papoulis, *Signal Analysis*, vol. 191 (McGraw-Hill, New York, 1978)
3. G Cherubini, E Eleftheriou, S Olcer, Filtered multitone modulation for very high-speed digital subscriber lines. IEEE J. Sel. Areas Commun. **20**(5), 1016–1028 (2002)
4. S Ariyavisitakul, A decision feedback equalizer with time-reversal structure. IEEE J. Sel. Areas Commun. **10**(3), 599–613 (1992)

5. JK Nelson, AC Singer, U Madhow, C McGahey, BAD: bidirectional arbitrated decision-feedback equalization. IEEE Trans. Commun. **53**(2), 214–218 (2005)
6. CW Wong, JM Shea, Y Lee, Hard-and soft-output trellis-based conflict resolution for bidirectional decision feedback equalization. IEEE Trans. Wireless Commun. **8**(7), 3780–3788 (2009)
7. P Kyritsi, P Stoica, G Papanicolaou, P Eggers, A Oprea, Time reversal and zero-forcing equalization for fixed wireless access channels, in *Conference Record of the Thirty-Ninth Asilomar Conference on Signals, Systems and Computers, 2005* (Pacific Grove, 29 October to 1 November 2005), pp. 1297–1301
8. J Choi, A bi-directional zero-forcing BLAST receiver. IEEE Trans. Signal Process. **52**(9), 2670–2673 (2004)
9. F Petré, G Leus, L Deneire, M Engels, M Moonen, H De Man, Space-time block coding for single-carrier block transmission DS-CDMA downlink. IEEE J. Sel. Areas Commun. **21**(3), 350–361 (2003)
10. G Fettweis, M Krondorf, S Bittner, GFDM - Generalized frequency division multiplexing, in *IEEE Semiannual Vehicular Technology Conference, VTC'09-Spring* (Barcelona, 26–29 April 2009), pp. 1–4
11. AV Oppenheim, RW Schafer, *Discrete-Time Signal Processing*, 3rd edn. (Prentice-Hall, Upper Saddle River, 2010)
12. J Proakis, M Salehi, *Digital Communications*, 5th edn. (McGraw-Hill, New York, 2008)

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